London Hopper Colloquium 2020

Research Spotlight Competition

Constructing Simple and Mutual Inductive Types

Stefania Damato

October 2020

School of Computer Science

University of Nottingham

supervised by Prof. Thorsten Altenkirch

psxsd7@nottingham.ac.uk

www.stefaniatadama.com

Problem and Motivation

Martin-Löf type theory – a formal language developed on the principles of constructive mathematics. We investigate the notion of an **inductive type**.

```
data ℕ : Set where
    zero : ℕ
    suc  : ℕ → ℕ
```

We want to:

- give a **complete specification** of simple and mutual inductive types
- **reduce** simple and mutual inductive types to **W-types**

in order to:

- achieve a mathematically **rigorous** system
- increase **reliability** of software by minimising our assumptions.

Related Work

Researchers have:

- proposed a general, mathematical **criterion** for the reduction.
- shown **analogous** results for other inductive types.

Approach and Uniqueness

- **Address a gap** in the formalisation of inductive types.
- Define constructions in **Agda**:
  - programming language and proof assistant
  - implements a version of Martin-Löf type theory
  - code in Agda is not run but **type checked.**
- **Constructively formalise** mathematical results in Agda.

Results and Contributions

- Constructed a '**theory of signatures**', a framework for expressing simple and mutual inductive types.
- Given any signature, we derived the **initial algebra** of the corresponding type.
- Provided a significant starting point for **reducing** simple and mutual inductive types to W-types.

- Advances the long-term goal of creating a small as possible **trusted code base**.

Future Work

Generalisation to more general inductive types e.g. nested inductive types, inductive families, inductive-inductive types.